### A. Practice

#### Simplifying Fractions

Example: Reduce 27/36

\[
\frac{27}{36} = \frac{9 \cdot 3}{9 \cdot 4} = \frac{9}{9} \cdot \frac{3}{4} = 1 \cdot \frac{3}{4} = \frac{3}{4}
\]

(Note that you must be able to find a common factor—in this case 9—in both the top and the bottom in order to reduce.)

1. \(\frac{13}{52} = \frac{3 + 6}{3 + 9} = \frac{3}{9} = \frac{1}{3}
\)

2. \(\frac{26}{65} = \frac{2}{5}
\)

#### Equivalent Fractions

Example: 3/4 is equivalent to how many eighths?

\[
\frac{3}{4} = \frac{3}{4} \cdot \frac{2}{2} = \frac{6}{8}
\]

4 to 5- Complete:

\[
\frac{4}{9} = \frac{5}{20}
\]

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

Example: 5/6 and 8/15

First find LCM of 6 and 15:

\[
\begin{align*}
6 & = 2 \cdot 3 \\
15 & = 3 \cdot 5 \\
\text{LCM} & = 2 \cdot 3 \cdot 5 = 30, \text{so}
\end{align*}
\]

\[
\frac{5}{30}, \text{ and } \frac{8}{16} \\
\frac{6}{30} = \frac{15}{30}
\]

1 to 7- Find equivalent fractions with the LCD:

6. \(\frac{2}{3} \text{ and } \frac{2}{9}
\)

7. \(\frac{1}{8} \text{ and } \frac{7}{12}
\)

8. Which is larger, 5/7 or 3/47?

(Hint: Find LCD fractions)

### B. Math Facts

If denominators are different, find equivalent fractions with common denominators, then proceed as before:

Example:

\[
\begin{align*}
\frac{4}{5} + \frac{2}{3} & = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} = 1 \frac{7}{15} \\
\frac{1}{2} + \frac{3}{4} & = \frac{3}{6} + \frac{6}{6} = \frac{9}{6} = 1 \frac{1}{6}
\end{align*}
\]

12. \(\frac{3}{5} + \frac{2}{3} = \frac{5 \cdot 3 + 2 \cdot 5}{15} = \frac{19}{15}
\)

13. \(\frac{5}{3} + \frac{1}{8} = \frac{5 \cdot 8 + 1 \cdot 3}{24} = \frac{43}{24}
\)

Multiplying fractions: multiply the tops, multiply the bottoms, reduce if possible.

Example:

\[
\begin{align*}
\frac{3}{4} \cdot \frac{2}{5} & = \frac{3 \cdot 2}{4 \cdot 5} = \frac{6}{20} = \frac{3}{10}
\end{align*}
\]

14. \(\frac{2}{3} \cdot \frac{3}{8} = \frac{3}{4}
\)

16. \(\frac{3}{2} \cdot \frac{2}{3} = \frac{9}{4}
\)

15. \(\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}
\)

17. \(\frac{1}{2} \cdot \frac{2}{1} = \frac{1}{2}
\)

Dividing fractions: a nice way to do this is to make a compound fraction and then multiply the top and bottom (of the big fraction) by the LCD of both:

Example:

\[
\begin{align*}
\frac{3}{4} \div \frac{2}{3} & = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} \\
\frac{2}{3} \div \frac{1}{2} & = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3}
\end{align*}
\]

Example:

\[
\begin{align*}
\frac{1}{2} - \frac{1}{3} & = \frac{3}{6} - \frac{2}{6} = \frac{1}{6} \\
\frac{1}{3} \div \frac{1}{2} & = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}
\end{align*}
\]

### C. Positive integer exponents and square roots of perfect squares

#### Meaning of exponents (powers):

Example: \(a^m = a \cdot a \cdot a \cdot \ldots \cdot a \) (m times)

Example: \(4^3 = 4 \times 4 \times 4 = 64\)

35 to 44-- Find the value:

35. \(3^2 =
\)

36. \((-3)^2 =
\)

37. \((-3)^3 =
\)

38. \(-3^2 =
\)

39. \((-2)^3 =
\)

40. \(100^2 =
\)

41. \((2.1)^2 =
\)

42. \((-.1)^3 =
\)

43. \((\frac{2}{3})^3 =
\)

44. \((-\frac{2}{3})^3 =
\)
Algebra Readiness Diagnostic Test Practice
Topic 6: Geometry (GEOM)

Formulas for perimeter P and area A of rectangles, squares, parallelograms, and triangles:

Rectangle with base b and altitude (height) h:

\[ P = 2b + 2h \]
\[ A = bh \]

If a wire is bent in this shape, the perimeter P is the length of the wire, and the area A is the number of square units enclosed by the wire.

**Example:** A rectangle with \( b = 7 \) and \( h = 8 \):
\[ P = 2b + 2h = 2 \cdot 7 + 2 \cdot 8 = 14 + 16 = 30 \text{ units} \]
\[ A = bh = 7 \cdot 8 = 56 \text{ square units} \]

A square is a rectangle with all sides equal, so the rectangle formulas apply (and simplify). If the side length is s:

\[ P = 4s \]
\[ A = s^2 \]

**Example:** A square with side \( s = 11 \) cm has
\[ P = 4s = 4 \times 11 = 44 \text{ cm} \]
\[ A = s^2 = 11^2 = 121 \text{ cm}^2 \text{ (sq.cm)} \]

A parallelogram with base b and height h and other side a:

\[ A = bh \]
\[ P = 2a + 2b \]

1. to 8: Find P and A:
   1. Rectangle with sides 5 and 10.
   2. Rectangle, sides 1.5 and 4.
   3. Square with side 3 miles.
   4. Square, side \( \frac{3}{4} \) yard.
   5. Parallelogram with sides 36 and 24, and height 10 (on side 36).
   6. Parallelogram, all sides 12, altitude 6.
   7. Triangle with sides 5, 12, 13. Side 5 is the altitude on side 12.
   8. Triangle shown:

**Formulas for circumference C and area A of a circle:**

A circle with radius \( r \) (and diameter \( d = 2r \)) has distance around (circumference)
\[ C = \pi d = 2\pi r \]

If a piece of wire is bent into a circular shape, the circumference is the length of the wire.

**Example:** A circle with radius \( r = 70 \) has \( d = 2r = 140 \) and exact circumference
\[ C = 2\pi r = 2 \cdot \pi \cdot 70 = 140\pi \text{ units} \]

If \( \pi \) is approximated by \( \frac{22}{7} \),
\[ C = 140\pi = 140(\frac{22}{7}) = 440 \text{ units (approx.)} \]

If \( \pi \) is approximated by 3.1,
\[ C = 140(3.1) = 434 \text{ units} \]

17. How many times the \( P \) and \( A \) of a 3 cm square are the \( P \) and \( A \) of a square with sides all 6 cm?

18. A rectangle has area 24 and one side 6. Find the perimeter.

19 to 20: A square has perimeter 30.

19. How long is each side?
20. What is its area?

21. A triangle has base and height each 7. What is its area?

**Pythagorean Theorem**

In any triangle with a 90° (right) angle, the sum of the squares of the legs equals the square of the hypotenuse.

(The legs are the two shorter sides; the hypotenuse is the longest side.)

If the legs have lengths \( a \) and \( b \), and \( c \) is the hypotenuse length, then
\[ a^2 + b^2 = c^2 \]

In words: "In a right triangle, leg squared plus leg squared equals hypotenuse squared."

**Example:** A right triangle has hypotenuse 5 and one leg 3. Find the other leg.

Since \( a^2 + b^2 = c^2 \),
\[ 3^2 + x^2 = 5^2 \]
\[ 9 + x^2 = 25 \]
\[ x^2 = 25 - 9 = 16 \]
\[ x = \sqrt{16} = 4 \]
example: A parallelogram has sides 4 and 6. 5 is the length of the altitude perpendicular to the side 4.

\[ P = 2a + 2b = 2 \cdot 6 + 2 \cdot 4 = 12 + 8 = 20 \text{ units} \]
\[ A = bh = 4 \cdot 5 = 20 \text{ square units} \]

In a triangle with side lengths a, b, and c, and altitude h to side b:

\[ P = a + b + c \]
\[ A = \frac{1}{2}bh = \frac{bh}{2} \]

example:

\[ P = a + b + c = 6 + 8 + 10 = 24 \text{ units} \]
\[ A = \frac{1}{2}bh = \frac{1}{2}(10 \cdot 4) = 20 \text{ sq. units} \]

The area of a circle is \( A = \pi r^2 \)

example: If \( r = 8 \), exact area is
\[ A = \pi r^2 = \pi \cdot 8^2 = 64\pi \text{ sq. units} \]

9 to 11: Find the exact \( C \) and \( A \) for a circle with:

9. radius \( r = 5 \) units
10. \( r = 10 \) feet
11. diameter \( d = 4 \) km

12 to 14: A circle has area 49\(\pi\).

12. What is its radius length?
13. What is the diameter?
14. Find its circumference.

15 to 16: A parallelogram has area 48 and two sides each of length 12.

15. How long is the altitude to those sides?
16. How long is each of the other two sides?

22 to 24: Find the length of the third side of the right triangle:
22. Leg 15, hypotenuse 17.
23. Hypotenuse 10, one leg 8.
24. Legs 5 and 12.

25 to 26: Find \( x \):

25.

26.

27. In right triangle \( ABC \), find \( AC \):

28. In \( \triangle RST \) with rt. angle \( R \), \( SR = 11 \) and \( TS = 61 \). Find \( RT \).

29. Would a triangle with sides \( 11 \) and \( 13 \) be a rt. triangle? Why or why not?
Similar triangles are triangles which are the same shape. If two angles of one triangle are equal respectively to two angles of another triangle, then the triangles are similar.

**Example:** \( \triangle ABC \) and \( \triangle DEF \) are similar:

- The pairs of sides which correspond are \( AB \) and \( DE \), \( BC \) and \( EF \), \( AC \) and \( DF \).
- \( \frac{AB}{DE}, \frac{BC}{EF}, \frac{AC}{DF} \).

**Example:**

30 to 32: Use this figure:

30. Find and name two similar triangles.
31. Draw the triangles separately and label them.
32. List the three pairs of corresponding sides.

If two triangles are similar, any two corresponding sides have the same ratio (fraction value):

**Example:**

33. Draw the similar triangles separately, label them, and write proportions for the corresponding sides.

34 to 37: Solve for \( x \):

**Example:**

38 to 45: Name the point with given coordinate:

46 to 51: On the number line above, what is the distance between the listed points? (Remember that distance is always positive.)

52 to 55: On the number line, find the distance from:

56 to 59: Draw a sketch to help find the coordinate of the point:

56. halfway between points with coordinates 4 and 14.
57. midway between \(-5\) and \(-1\).
58. which is the midpoint of the segment joining \(-8\) and 4.
59. on the number line the same distance from \(-6\) as it is from 10.

**Coordinate plane graphing**

To locate a point on the plane, an ordered pair of numbers is used, written in the form \((x, y)\).

60 to 63: Identify coordinates \( x \) and \( y \) in each ordered pair:

60. \((3, 0)\) 62. \((5, -2)\) 61. \((-2, 5)\) 63. \((0, 3)\)

To plot a point, start at the origin and make two moves, first in the \( x \)-direction (horizontal) and then in the \( y \)-direction (vertical) indicated by the ordered pair.

**Example:** Plot \((-3, 4)\):

(a) Start at the origin, move left 3 (since \( x = -3 \)).
(b) Then (from there), up 4 (since \( y = 4 \)).
(c) Put a dot there to indicate the point \((-3, 4)\).

64. On graph paper, join these points in order: \((-3, -2), (1, -4), (3, 0), (2, 3), (-1, 2), (3, 0), (-3, -2), (-1, 2), \) and \((1, -4)\).

65. Two of the lines you drew in problem 64 cross each other. What are the coordinates of the crossing point?

66. In what quadrant is the point \((a, b)\) if \( a > 0 \) and \( b < 0 \)?

67 to 69: \( ABCD \) is a square, with \( C(5, -2) \) and \( D(-1, -2) \). Find:

67. the length of each side.
68. the coordinates of \( A \).
69. the coordinates of the midpoint of \( \overline{DC} \).

70 to 72: Given \( A(0, 5), B(12, 0) \), find its length.
71. Find the midpoint of \( \overline{AB} \) and label it \( C \). Find the coordinates of \( C \).
72. What is the area of the triangle formed by \( A, B, \) and the origin?
Answers:

1. 30 unis,
   50 unis²
   (unis² means square unis)
2. 11 un., 6 un.²
3. 1 1/2 mi., 6 mi²
4. 3 yd², 1/2 yd²
5. 120 un.,
   360 un.²
6. 46 un.,
   72 un.²
7. 30 un.,
   30 un.²

8. 12 un., 6 un.²
9. 10π un.,
   25π un.²
10. 20π ft,
    100π ft²
11. 4π km²
    4π km²
12. 7
13. 14
14. 14π
15. 4
16. Can't tell
17. P is 2 times,
    A is 4 times
18. 20
19. 7 1/4
20. 11/4
21. 24 1/4
22. 8
23. 6
24. 13
25. 9
26. 41
27. 10
28. 60
29. No, because
   \(7^2 + 11^2 = 13^2\)
30. \(\triangle ABC\)
    \(\triangle ACD\)
31. 

32. \(\triangle ABC\)
    \(\triangle ACD\)
33. \(\frac{3}{2} = \frac{15}{12}\)
34. \(\frac{49}{1} = \frac{24}{1}\)
35. \(\frac{24}{1} = \frac{3}{2} = \frac{4}{1}\)
36. \(\frac{2}{1}\)
37. \(\frac{40}{7}\)
38. \(\frac{40}{7}\)
39. \(\frac{40}{7}\)
40. \(\frac{40}{7}\)
41. \(\frac{40}{7}\)
42. \(\frac{40}{7}\)
43. \(\frac{40}{7}\)
44. \(\frac{40}{7}\)
45. \(\frac{40}{7}\)
46. 2.75
47. 2
48. 3
49. 1
50. 2
51. \(\frac{1}{2}\)
52. 3
53. 11
54. 11
55. 3
56. 6
57. -3
58. -2
59. 2
60. \(x = 3, \ y = 0\)
61. \(x = -2, \ y = 5\)
62. \(x = 5, \ y = -2\)
63. \(x = 0, \ y = 3\)
64. Can draw in one continuous broken line, no reversing?
65. (0, -1)
66. IV
67. 6
68. (-1, 4)
69. (2, -2)
70. 13
71. (6, 1)
72. 30

6: GEOM
### Algebra Readiness Diagnostic Test Practice

**Topic 4: Exponents (EXPS)**

**Directions:** Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

#### Positive integer exponents

**Meaning of exponents:**

| example: | $3^4 = 3 \times 3 \times 3 \times 3$  
| example: | $= 3 \cdot 3 \cdot 3 \cdot 3 = 81$  
| example: | $4^3 = 4 \cdot 4 \cdot 4 = 64$  

1 to 12: Find the value:

1. $3^2 = $
2. $2^3 = $
3. $(-3)^2 = $
4. $-(3)^2 = $
5. $-3^2 = -(3^2) = $
6. $-2^3 = $
7. $(-2)^3 = $
8. $100^2 = $
9. $(2.1)^2 = $
10. $(-.1)^3 = $
11. $(\frac{2}{3})^3 = $
12. $(\frac{-2}{3})^3 = $

\(a^b\) means use \(a\) as a factor \(b\) number of times.

\((b\) is the exponent or power of \(a\))

### 25 to 30: Simplify:

| example: | $\frac{8}{2}^4 = \frac{8}{16} = \frac{1}{2}$  
| example: | $\frac{6^3}{6^2} = \frac{216}{36} = 6$  

25. $\frac{6}{3^2} = $
26. $\frac{2^3}{8} = $
27. $\frac{4 \times 5}{10} = $
28. $\frac{10}{4^2 \times 5} = $
29. $\frac{2^3 \cdot 2^4}{2^3 \cdot 2} = $
30. $\frac{5 \times 12}{6^2 \times 10} = $

31 to 38: Find the value:

31. $3^2 + 4^2 = $
32. $5^3 = $
33. $3^2 + 4^2 + 12^2 = $
34. $13^2 = $
35. $(3.1)^2 - (0.3)^3 = $
36. $(3.1)^2 + (0.3)^3 = $
37. $3^3 + 4^3 + 5^3 = $
38. $6^3 = $

#### 47 to 56:

47. $4^1 \times 4^2 = $
48. $5^1 \cdot 5^3 = $
49. $3^3 \cdot 3^1 = $
50. $-(1)^3 \times (-1)^4 = $
51. $10 \times 10^6 = $
52. $10 \times 10^4 = $
53. Make a formula by filling in the brackets: $a^b \cdot a^c = a^{b+c}$

This is an exponent rule.

### 54 to 56:

54. $3^4 = $
55. $3^4 = $
56. $729 + 81 = $  

**note:** $3^4 + 3^4 = \frac{3^6}{3^4} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \frac{3}{3} = 3^2$

57. Circle the exponents: $\frac{3^6}{3^4} = 3^2$

58. How are the circled numbers related?

59 to 63: Write each expression as a power.
example: $2^3$ means $2 \cdot 2 \cdot 2 \cdot 2$
$2^3$ has value 32
5 is the exponent or power
2 is the factor
example: $5 \cdot 5$ can be written $5^2$
Its value is 25
example: $4^1 = 4$

13 to 24: Write the meaning and find the value:

13. $6^3 =$
14. $(-4)^2 =$
15. $0^4 =$
16. $7^1 =$
17. $1^6 =$
18. $(-1)^3 =$
19. $(0.1)^4 =$
20. $(\frac{3}{4})^4 =$
21. $(1\frac{1}{2})^2 =$
22. $2^{10} =$
23. $0.03^2 =$
24. $3^2 \cdot 2^3 =$

Integer exponent laws

39 to 40: Write the meaning
(not the value):

39. $3^3 =$
40. $3^4 =$

41. Write as a power of 3:
$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 =$
42. Write the meaning: $3^2 \cdot 3^4 =$
43. Write your answer to 42 as a power of 3, then find the value.
44. Now find each value and solve: $3^2 \cdot 3^6 =$
45. So $3^2 \cdot 3^4 = 3^6$. Circle each of the powers. Note how the circled numbers are related.
46. How are they related?

47 to 52: Write each expression as a power of the same factor:

example: $3^2 \cdot 3^4 = 3^6$

example: $\frac{3^6}{3^3} = 3^3$

59. $2^4 + 2^4 =$
60. $\frac{2^2}{2} =$
61. $\frac{5^2}{5} =$
62. $\frac{(-4)^7}{(-4)^2} =$
63. $\frac{1^3}{1^3} =$

64. Make a formula by filling in the brackets: $\frac{a^b}{a^c} = a^{(}$
This is another exponent rule.

65 to 67: Find each value:

65. $4^3 =$
66. $4^6 =$
67. $(4^3)^2 = (64)^2 =$
68 to 69: Write the meaning of each expression:

**Example:** 
\((4^3)^2 = 4^6\)
\[= 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^6\]

68. \((3^2)^4 = \)

69. \((5^1)^3 = \)

70. Circle the three exponents:
\((4^3)^2 = 4^6\)

71. What is the relation of the circled numbers?

72. Make a rule: \((a^b)^c = a^{bc}\)

73. Write your three exponent rules in the box:

1. \(a^b \cdot a^c = a^{b+c}\)
2. \(\frac{a^b}{a^c} = a^{b-c}\)
3. \((a^b)^c = a^{bc}\)

**Example:**
32800 = 3.2800 \(\times 10^4\)
if the zeros in the ten’s and one’s places are significant. If the one’s zero is not significant, write 3.28 \(\times 10^4\) if neither is significant: 3.28 \(\times 10^4\)

**Example:**
\[0.004031 = 4.031 \times 10^{-3}\]
\[2 \times 10^2 = 200\]
\[9.9 \times 10^{-1} = 0.99\]

81 to 84: Write in scientific notation:

- 81. 93,000,000 =
- 82. 0.000042 =
- 83. 5.07 =
- 84. –32 =

85 to 87: Write in standard notation:

- 85. 1.4030 \(\times 10^3\) =
- 86. 9.11 \(\times 10^{-2}\) =
- 87. 4 \(\times 10^{-4}\) =

92. \(1.8 \times 10^8 = \frac{3.6 \times 10^7}{2}\)
93. \((4 \times 10^3)^2 = \)
94. \((1.5 \times 10^2) \times (5 \times 10^3) = \)
95. \((1.25 \times 10^3) \times (4 \times 10^{-2}) = \)

Square roots of perfect squares

\(\sqrt{a} = b\) means \(b^2 = a\), where \(b \geq 0\)
Thus \(\sqrt{49} = 7\), because \(7^2 = 49\).
Also, \(-\sqrt{49} = -7\).

Note: \(\sqrt{49}\) does not equal \(-7\),
even though \((-7)^2\) does = 49) because \(-7\) is not \(\geq 0\)

**Example:**
If \(\sqrt{a} = 10\), then \(a = 100\),
because \(10^2 = a = 100\)

96 to 99: Find the value and tell why:

- 96. If \(\sqrt{a} = 5\), then \(a = \)
- 97. If \(\sqrt{x} = 4\), then \(x = \)
- 98. If \(\sqrt{36} = b\), then \(b = \)
- 99. If \(\sqrt{169} = y\), then \(y = \)
74 to 80: Use the rules to write each expression as a power of the factor, and tell which rule you're using:

74. \(3^4 \cdot 3^6 = \)

75. \(\frac{2^{10}}{2^5} = \)

76. \((2^4)^2 = \)

77. \((3^4)^4 = \)

78. \(\frac{3^4}{3} = \)

79. \((5^2)^3 = \)

80. \(10^4 \cdot 10^3 = \)

Scientific notation

Note that scientific form always looks like \(a \times 10^n\), where \(1 \leq a < 10\), and \(n\) is an integer power of 10.

To compute with numbers written in scientific form, separate the parts, compute, then recombine:

**Example:**

\[
(3.14 \times 10^5)(2) = 6.28 \times 10^5
\]

\[
\frac{4.28 \times 10^6}{2.14 \times 10^2} = 2.00 \times 10^4
\]

88 to 95: Write answer in scientific notation:

88. \(10^{40} \times 10^2 = \)

89. \(\frac{10^{40}}{10^{10}} = \)

90. \(\frac{1.86 \times 10^6}{3 \times 10} = \)

91. \(3.6 \times 10^5 = \)

100 to 110: Find the value:

100. \(\sqrt{81} = \)

101. \(8^2 = \)

102. \(\sqrt{8} = \)

103. \(\sqrt[-3]{-7} = \)

104. \(\sqrt{6^2 + 8^2} = \)

105. \(\sqrt[3]{3^2 + 4^2} = \)

106. \(\sqrt[3]{3^2 + 4^2 + 12^2} = \)

107. \(\sqrt{17^2 - 15^2} = \)

108. \(\sqrt{13^2 - 12^2} = \)

109. \(\sqrt[5]{5} = \)

110. \(\sqrt[7]{7} = \)

Answers:

| 1. | 9 |
| 2. | 8 |
| 3. | 9 |
| 4. | -9 |
| 5. | -9 |
| 6. | -8 |
| 7. | -8 |
| 8. | 10000 |
| 9. | 4.41 |
| 10. | -0.001 |
| 11. | \(\frac{1}{3}\) |
| 12. | \(-\frac{1}{3}\) |
| 13. | 6.6 - 6 = 216 |
| 14. | \((-4)(-4) = 16 |
| 15. | 0 \cdot 0 \cdot 0 = 0 |
| 16. | 7 = 7 |
| 17. | 1 - 1 - 1 = 1 |
| 18. | -(-1)(-1)(-1) = -1 |
| 19. | \((-1)(1)(1)(1) = 0.0001 |

| 20. | \(\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27} \)
| 21. | \(\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1 \)
| 22. | \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 \)
| 23. | \((0.03)(0.03) = 0.0009 \)
| 24. | \(\frac{3}{3} \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 72 \)
| 25. | \(\frac{2}{3} \)
| 26. | 4 |
| 27. | 2 |
| 28. | \(\frac{2}{3} \)
| 29. | 2 |
| 30. | \(\frac{1}{2} \)
| 31. | 25 |
| 32. | 25 |
| 33. | 169 |
| 34. | 169 |
| 35. | 9.6091 |
| 36. | 6.6109 |
| 37. | 216 |
| 38. | 216 |
| 39. | 3.3 |
| 40. | 3.3 - 3.3 |
| 41. | 3.6 |
| 42. | 3.3 |
| 43. | 3.6 |
| 44. | 4.91 - 172 |
| 45. | 5.3(2) - 3(4) |
| 46. | \(2 + 4 = 6 \)
| 47. | \(4^2 \)
| 48. | 5^6 |
| 49. | \(\frac{3}{3} \)
| 50. | \(\frac{1}{3} \)
| 51. | 10^3 |
| 52. | 10^2 |
| 53. | \(a^b \cdot a^c = a^{b+c} \)
| 54. | 729 |
| 55. | 81 |
| 56. | 9 |
| 57. | \(\frac{3}{3} \)
| 58. | 6 - 4 = 2 |
| 59. | \(2^3 \)
| 60. | \(2^4 \)
| 61. | \(5^1 \)
| 62. | \((-4)^3 \)
| 63. | \(1^2 \text{ or any power of 1} \)
| 64. | \(a^b \cdot a^c = a^{b+c} \)
| 65. | 64 |
| 66. | 4096 |
| 67. | 4096 |
| 68. | \(3^2 \cdot 3^3 \cdot 3^2 = 3^9 \)
| 69. | 5 \cdot 3 \cdot 5 |
| 70. | \((a^3)^2 \)
| 71. | 3 \times 3 = 9 |
| 72. | \((a^4)^6 \)
| 73. | \(a^b \cdot a^c = a^{b+c} \)
| 74. | \(3^{10} \text{ rule 1} \)
| 75. | \(2^{10} \text{ rule 2} \)
| 76. | \(2^{10} \text{ rule 3} \)
| 77. | \(3^{14} \text{ rule 3} \)
| 78. | 3^7 \text{ rule 2} \)
| 79. | 5^2 \text{ rule 3} \)
| 80. | 10^7 \text{ rule 1} \)
| 81. | 9.3 \times 10^7 |
| 82. | 4.2 \times 10^5 |
| 83. | 5.07 |
| 84. | 3.2 \times 10 |
| 85. | 1403.0 |
| 86. | 0.9911 |
| 87. | 0.000004 |
| 88. | 10^{42} |
| 89. | 10^{30} |
| 90. | 6.2 \times 10^2 |
| 91. | 2.0 \times 10^2 |
| 92. | 5.0 \times 10^2 |
| 93. | 1.6 \times 10^7 |
| 94. | 7.5 \times 10^3 |
| 95. | 5 |
| 96. | 25 \cdot 5^2 = 25 |
| 97. | 18 \cdot 4^2 = 14 |
| 98. | 6 \cdot 6^2 = 36 |
| 99. | 13 |
| 100. | 9 |
| 101. | 64 |
| 102. | 8 |
| 103. | 7 |
| 104. | 10 |
| 105. | 5 |
| 106. | 13 |
| 107. | 8 |
| 108. | 5 |
| 109. | 8 |
| 110. | 9 |
### A. Positive Integer Exponents

$a^b$ means use $a$ as a factor $b$ times. ($b$ is the exponent or power of $a$.)

#### Example:

2^3 means $2 \cdot 2 \cdot 2 \cdot 2$, and has value 32.

Example: $c \cdot c \cdot c = c^3$

1 to 14: Find the value:

1. $2^3 = $  
2. $3^3 = $  
3. $-4^2 = $  
4. $(-4)^2 = $  
5. $0^4 = $  
6. $1^4 = $  
7. $\left(\frac{2}{3}\right)^4 = $  
8. $(.2)^3 = $  
9. $\left(1 \frac{1}{2}\right)^2 = $  
10. $2^{10} = $  
11. $(-2)^9 = $  
12. $\left(2 \frac{2}{3}\right)^2 = $  
13. $(-1.1)^3 = $  
14. $3^2 \cdot 2^3 = $  

**Example:** Simplify: $a \cdot a \cdot a \cdot a \cdot a = a^5$

15 to 18: Simplify:

15. $3^2 \cdot x^4 = $  
16. $2^4 \cdot b \cdot b \cdot b = $  
17. $4^2 (-x)(-x)(-x) = $  
18. $(-y)^4 = $  
19 to 28: Find $x$:

19. $2^3 \cdot 2^4 = 2^x$  
20. $2^3 \cdot 2^4 = 2^x$  
21. $3^{-4} = \frac{1}{3^x}$  
22. $\frac{2^2}{5^2} = 5^x$  
23. $(2^3)^4 = 2^x$  
24. $8 = 2^x$  
25. $a^3 \cdot a = a^x$  
26. $\frac{b^{10}}{b^x} = b^x$  
27. $\frac{c^x}{c^4} = c^x$  
28. $\frac{a^{xy}}{a^{xy}} = a^x$  

29 to 41: Find the value:

29. $7 \cdot x^0 = $  
30. $3^{-4} = $  
31. $2^5 \cdot 2^4 = $  
32. $0^5 = $  
33. $5^0 = $  
34. $(-3)^3 - 3^3 = $  
35. $x^{x+3} \cdot x^{x-3} = $  
36. $x^{x+3} \cdot x^{x-3} = $  
37. $2x^{-3} = $  
38. $(a^{x+3})^{x-3} = $  
39. $(x^3)^2 = $  
40. $(3x^2)^2 = $  
41. $(-2xy^3)^3 = $  

Note that scientific form always looks like $a \times 10^n$ where $1 \leq a < 10$, and $n$ is an integer power of 10.

42 to 45: Write in scientific notation:

42. $93,000,000 = $  
43. $0.0000042 = $  
44. $5.07 = $  
45. $-32 = $  

46 to 48: Write in standard notation:

46. $1.4030 \times 10^3 = $  
47. $-9.11 \times 10^2 = $  
48. $4 \times 10^{-5} = $  

To compute with numbers written in scientific form, separate the parts, compute, then recombine.

**Example:**

$\frac{(3.14 \times 10^3)(2)}{2} = \frac{(3.14)(2) \times 10^3}{2} = 6.28 \times 10^3$

**Example:**

$\frac{4.28 \times 10^6}{2.14 \times 10^2} = \frac{4.28}{2.14} \times 10^{6-2} = 2.00 \times 10^4$

**Example:**

$\frac{2.01 \times 10^{-3}}{8.04 \times 10^{-4}} = \frac{.250 \times 10^{-3}}{2.50 \times 10^2}$
### B. Integer Exponents

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>$a^b \cdot a^c = a^{b+c}$</td>
</tr>
<tr>
<td>II.</td>
<td>$\frac{a^b}{a} = a^{b-1}$</td>
</tr>
<tr>
<td>III.</td>
<td>$(a^b)^c = a^{bc}$</td>
</tr>
<tr>
<td>IV.</td>
<td>$(ab)^c = a^c \cdot b^c$</td>
</tr>
<tr>
<td>V.</td>
<td>$(\frac{a}{b})^c = \frac{a^c}{b^c}$</td>
</tr>
<tr>
<td>VI.</td>
<td>$a^b - 1$ (if $a \neq 0$)</td>
</tr>
<tr>
<td>VII.</td>
<td>$a^{-c} = \frac{1}{a^c}$</td>
</tr>
</tbody>
</table>

### C. Scientific Notation

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>example: $32500$ = $3.2800 \times 10^4$ if the zeros in the ten's and one's places are significant. If the one's zero is not, write $3.280 \times 10^3$; if neither is significant: $3.28 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>example: $0.004031 = 4.031 \times 10^{-3}$</td>
</tr>
<tr>
<td>example: $2 \times 10^2 = 200$</td>
<td></td>
</tr>
<tr>
<td>example: $9.9 \times 10^{-1} = 0.99$</td>
<td></td>
</tr>
</tbody>
</table>

49 to 56 -- Write answer in scientific notation:

49. $10^{-40} \times 10^{-2} =$

50. $\frac{10^{-40}}{10^{-10}} =$

51. $1.86 \times 10^{-4} =$

52. $3.6 \times 10^{-5} =$

53. $1.8 \times 10^{-8} =$

54. $(4 \times 10^{-3})^2 =$

55. $(2.5 \times 10^{-2})^{-1} =$

56. $(-2.92 \times 10^{-3})(4.1 \times 10^{-7}) =$

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D. Simplification of square roots

\[ \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \]  if a and b are both non-negative (a \geq 0 and b \geq 0).

Example:
\[ \sqrt{32} - \sqrt{16} - \sqrt{2} = 4\sqrt{2} \]

Example:
\[ \sqrt{75} = \sqrt{5} \cdot \sqrt{25} = \sqrt{5} \cdot 5 = 5\sqrt{5} \]

Example:
If \( x \geq 0 \), \( \sqrt{x^6} = x^3 \)
If \( x < 0 \), \( \sqrt{x^6} = |x^3| \)

Note: \( \sqrt{a} = \frac{b}{c} \) means (by definition) that
1) \( b = a \), and
2) \( b \geq 0 \)

E. Multiplying Square Roots

\[ \sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \]
if \( a \geq 0 \) and \( b \geq 0 \)

Example:
\[ \sqrt{6} \cdot \sqrt{24} = \sqrt{6 \cdot 24} = \sqrt{144} = 12 \]

Example:
\[ \sqrt{2} \cdot \sqrt{6} = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3} \]

Example:
\[ \left(5\sqrt{2}\right)\left(3\sqrt{2}\right) = 15\sqrt{4} = 15 \cdot 2 = 30 \]

74 to 79-- Simplify:

74. \( \sqrt{3} \cdot \sqrt{3} = \)
75. \( \sqrt{5} \cdot \sqrt{4} = \)
76. \( \left(2\sqrt{3}\right)\left(3\sqrt{2}\right) = \)
77. \( \left(\sqrt{9}\right)^2 = \)
78. \( \left(\sqrt{5}\right)^2 = \)
79. \( \left(\sqrt{3}\right)^2 = \)

80 to 81-- Find the value of \( x \):

80. \( \sqrt{4} \cdot \sqrt{9} = \sqrt{x} \)
81. \( 3\sqrt{2} \cdot \sqrt{5} = 3\sqrt{x} \)

87 to 94-- Simplify:

87. \( \sqrt{\frac{9}{4}} = \)
88. \( \sqrt{\frac{18}{9}} = \)
89. \( \frac{4}{9} = \)
90. \( \frac{3}{2} = \)
91. \( \frac{1}{\sqrt{5}} = \)
92. \( \frac{3}{\sqrt{3}} = \)
93. \( \frac{\sqrt{a}}{\sqrt{b}} = \)
94. \( \sqrt{2} + \frac{1}{\sqrt{2}} = \)
G. Dividing Square Roots

\[ \sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \]

if \( a \geq 0 \) and \( b > 0 \)

Example:

\[ \sqrt{2} \div \sqrt{64} = \frac{\sqrt{2}}{\sqrt{64}} = \frac{\sqrt{2}}{8} \text{ (or } \frac{1}{\sqrt{2}} \text{)} \]

82 to 86—Simplify:

82. \( \sqrt{3} + \sqrt{4} = \)

83. \( \frac{\sqrt{9}}{25} = \)

84. \( \frac{\sqrt{49}}{2} = \)

85. \( \sqrt{36} + 4 = \)

86. \( \frac{-8}{\sqrt{16}} = \)

If a fraction has a square root on the bottom, it is sometimes desirable to find an equivalent fraction with no root on the bottom. This is called rationalizing the denominator.

Example:

\[ \frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5} \cdot \sqrt{2}}{\sqrt{8} \cdot \sqrt{2}} = \frac{\sqrt{10}}{4} \]

Example:

\[ \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \]

E. Adding and subtracting square roots

Example:

\[ \sqrt{5} + 2\sqrt{5} = 3 \sqrt{5} \]

Example:

\[ \sqrt{32} - 4\sqrt{2} - \sqrt{2} = 3 \sqrt{2} \]

70 to 73—Simplify:

70. \( \sqrt{5} + \sqrt{5} = \)

71. \( 2\sqrt{3} + \sqrt{27} - \sqrt{75} = \)

72. \( 3\sqrt{2} + \sqrt{2} = \)

73. \( 5\sqrt{3} - \sqrt{3} = \)
**Topic 8: Geometric Measurement**

**Directions:** Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

**A. Intersecting lines and parallels**

If two lines intersect as shown, adjacent angles add to 180°. For example, \( a + d = 180° \).

- Non-adjacent angles are equal:
  - for example, \( a = c \).

If two lines, \( a \) and \( b \), are parallel and are cut by a third line, \( x \) as shown, then \( x = z \), \( w = z \), \( w + y = 180° \), so \( z + y = 180° \).

**Example:** If \( a = 3x \) and \( c = x \), find the measure of \( c \).

\[
b = c, \text{ so } b = x. \\
a + b = 180, \text{ so } 3x + x = 180, \\
giving \ 4x = 180, \text{ or } x = 45°. \\
Thus \ c = x = 45°.
\]

1. Given \( x = 127° \). Find the measure of the other angles.

1. \( t \)  
3. \( z \)  
2. \( y \)  
4. \( w \)  
5. Find \( x \):

**Example:** Find \( P \) and \( A \) for each of the following figures:

6. Rectangle with sides 5 and 10.
7. Rectangle, sides 1.5 and 4.
8. Square with side 3 mi.
9. Square, side \( \frac{3}{4} \) yd.
10. Parallelogram with sides 36 and 24, and height 10 (on side 36).
11. Parallelogram, all sides 12, altitude 6.
12. Triangle with sides 5, 12, 13, and 15 is the height on side 12.
13. The triangle shown:

**Example:** Parallelogram has sides 4 and 6, and 5 is the length of the altitude perpendicular to the side 4.

\[
\begin{align*}
P &= 2a + 2b = 2 \cdot 6 + 2 \cdot 4 = 20 \text{ units} \\
A &= bh = 4 \cdot 5 = 20 \text{ sq. units}
\end{align*}
\]

**D. Formulas for volume \( V \)**

A **rectangular solid** (box) with length \( l \), width \( w \), and height \( h \) has volume \( V = lwh \).

**Examples:**

- A box with dimensions 3, 7, and 11 has what volume?
  \( V = lwh = 3 \cdot 7 \cdot 11 = 231 \text{ cu. units} \)

- A **cube** is a box with all edges equal. If the edge is \( e \), the volume is \( V = e^3 \)

**Example:** A cube has edge 4 cm.

\( V = e^3 = 4^3 = 64 \text{ cm}^3 \) (cu. cm.)

A (right circular) **cylinder** with radius \( r \) and altitude \( h \) has

\( V = \pi r^2 h \)

**Example:** A cylinder has \( r = 10 \) and \( h = 14 \). The exact volume is \( V = \pi r^2 h = \pi \cdot 10^2 \cdot 14 = 1400 \text{ cu. units} \)

If \( \pi \) is approximated by \( \frac{22}{7} \), \( V = 1400\left(\frac{22}{7}\right) = 4400 \text{ cu. units} \).

If \( \pi \) is approximated by 3.14, \( V = 1400(3.14) = 4396 \text{ cu. units} \).

A **sphere** (ball) with radius \( r \) has volume \( V = \frac{4}{3} \pi r^3 \)

**Example:** The exact volume of a sphere with radius 6 in. is \( V = \frac{4}{3} \pi (6^3) = \frac{4}{3}(216) = 288 \pi \text{ in.}^3 \)
B. Formulas for perimeter \( P \) and area \( A \) of triangles, squares, rectangles, and parallelograms

**Rectangles**, base \( b \), altitude (height) \( h \):
\[
P = 2b + 2h
\]
\[
A = bh
\]
If a wire is bent in the shape, the perimeter is the length of the wire, and the area is the number of square units enclosed by the wire.

**Example:** Rectangles with \( b = 7 \) and \( h = 8 \):
\[
P = 2b + 2h = 2 \cdot 7 + 2 \cdot 8 = 14 + 16 = 30 \text{ units}
\]
\[
A = bh = 7 \cdot 8 = 56 \text{ sq. units}
\]

A **square** is a rectangle with all sides equal, so the formulas are the same (and simpler if the side length is \( s \)):
\[
P = 4s
\]
\[
A = s^2
\]

**Example:** Square with side 11 cm has
\[
P = 4s = 4 \cdot 11 = 44 \text{ cm}
\]
\[
A = s^2 = 11^2 = 121 \text{ cm}^2 \text{ (sq. cm)}
\]

A **parallelogram** with base \( b \) and height \( h \) has
\[
A = bh
\]
If the other side length is \( a \), then \( P = 2a + 2b \)

C. Formulas for circle Area \( A \) and Circumference \( C \)

A circle is with radius \( r \) (and diameter \( d = 2r \)) has distance around (circumference)
\[
C = \pi d \quad \text{or} \quad C = 2\pi r
\]

(If a piece of wire is bent into a circular shape, the circumference is the length of wire.)

**Example:** A circle with radius \( r = 70 \) has
\[
d = 2r = 140 \text{ and exact circumference}
\]
\[
C = 2\pi r = 2 \cdot \pi \cdot 70 = 140\pi \text{ units.}
\]

If \( \pi \) is approximated by \( \frac{22}{7} \),
\[
C = 140\pi = 140\left(\frac{22}{7}\right) = 440 \text{ units approximately.}
\]

If \( \pi \) is approximated by 3.1, the approximate \( C = 140(3.1) = 434 \text{ units} \).

The area of a circle is \( A = \pi r^2 \):

**Example:** If \( r = 8 \),
\[
A = \pi r^2 = \pi \cdot 8^2 = 64\pi \text{ sq. units}
\]

14 to 16—Find \( C \) and \( A \) for each circle:
14. \( r = 5 \) units
15. \( r = 10 \) feet
16. \( d = 4 \text{ km} \)

E. Sum of the interior angles of a triangle:
The three angles of any triangle add to 180°.

**Example:** Find the measures of angles \( C \) and \( A \):
\[
\angle C \text{ (angle C) is marked to show its measure is 90°.}
\]
\[
\angle B + \angle C = 36 + 90 = 126, \text{ so}
\]
\[
\angle A = 180 - 126 = 54°.
\]

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25 to 29—Given two angles of a triangle, find the measure of the third angle:

25. 30°, 90°
26. 115°, 36°
27. 90°, 17°

28. 82°, 82°
29. 68°, 44°

If two triangles are similar, any two corresponding sides have the same ratio (fraction value):

Example: the ratio $a$ to $x$,

or $\frac{a}{x}$, is the same as $\frac{b}{y}$

and $\frac{c}{z}$. Thus, $\frac{a}{x} = \frac{b}{y}$,

$\frac{c}{z}$, and $\frac{b}{y} = \frac{c}{z}$. Each of these equations is called a proportion.

16 of 18

If the sum of the squares of two sides of a triangle is the same as the square of the third side, the triangle is a right triangle.

Example:

Is a triangle with sides 20, 29, 21 a right triangle?

$20^2 + 21^2 = 29^2$

so it is a right triangle.

57 to 59—Is a triangle right, if it has sides:

57. 17, 8, 15
58. 4, 5, 6
59. 60, 61, 11
F. Isosceles Triangles

An isosceles triangle is defined to have at least two sides with equal measures. The equal sides may be marked:

or the measure may be given:

30 to 35—Is the triangle isosceles?
30. Sides 3, 4, 5
31. Sides, 7, 4, 7
32. Sides, 8, 8, 8

The angles which are opposite the equal sides also have equal measures (and all three angles add to 180°).

Example: Find the measure of \( \angle A \) and \( \angle C \), given \( \angle B = 65^\circ \):
\[
\angle A + \angle B + \angle C = 180^\circ, \text{ and}
\angle A = \angle B = 65^\circ, \text{ so } \angle C = 50^\circ
\]

36. Find measures of \( \angle A \) and \( \angle B \), if \( \angle C = 30^\circ \).

37. Find measures of \( \angle B \) and \( \angle C \), if \( \angle A = 30^\circ \).

38. Find measure of \( \angle A \).

39. If the angles of a triangle are 30°, 60°, and 90°, can it be isosceles?

40. If two angles of a triangle are 45° and 60°, can it be isosceles?

44 to 45: Write proportions for the two similar triangles:

44.

45.

Example: Find \( x \):
Write and solve a proportion:
\[
\frac{\frac{2}{3}}{x} = \frac{3}{x}, \text{ so } 2x = 15, x = 7\frac{1}{2}
\]

46 to 49—Find \( x \):

46.

47.

48.

49.

50. Find \( x \) and \( y \):
If a triangle has equal angles, the sides opposite these angles also have equal measures.

Example: Find the measure of ∠B, ∠C and ∠D. Given this figure, and ∠C = 40°:

∠B = 70° (because all angles add to 180°)

Since ∠A = ∠B, AC = BC = 16. AB can be found with trig later.

41. Can a triangle be isosceles and have a 90° angle?

42. Given ∠D = ∠E = 68° and DF = 6. Find the measure of ∠F and length of FE.

G. Similar Triangles:
If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

Example: △ABC and △FED are similar:

The pairs are corresponding sides are AB and FE, BC and ED, and AC and FD.

43. Name two similar triangles and list the pairs of corresponding sides.

H. Pythagorean Theorem
In any triangle with a 90° (right) angle, the sum of the squares of the legs equals the square of the hypotenuse. (The legs are the shorter sides; the hypotenuse is the longest side.) If the legs have lengths a and b, and the hypotenuse length is c, then

\[ a^2 + b^2 = c^2 \]

(In words, 'In a right triangle, leg squared plus leg squared equals hypotenuse squared.')

Example:
A right triangle has hypotenuse 5 and one leg 3. Find the length of the third side.

Since \[ a^2 + b^2 = c^2 \]

\[ 9 + x^2 = 25 \]
\[ x^2 = 25 - 9 = 16 \]
\[ x = \sqrt{16} = 4 \]

51 to 54—Each line of the chart lists two sides of a right triangle. Find the length of the third side:

<table>
<thead>
<tr>
<th>Leg</th>
<th>Leg</th>
<th>Hyp</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>\sqrt{2}</td>
<td>\sqrt{3}</td>
<td></td>
</tr>
</tbody>
</table>

55 to 56—Find x:

55.

56.
Elementary Algebra Diagnostic Test Practice
Topic 9: Word Problems

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Arithmetic, Percent, and Average

1. What is the number, which when multiplied by 32, gives 32 • 46?
2. If you square a certain number, you get 9\(^2\). what is the number?
3. What is the power of 36 that gives 36\(^3\)?
4. Find 3% of 36.
5. 55 is what percent of 88?
6. What percent of 55 is 88?
7. 45 is 80% of what number?
8. What is 8.3% of $7000?
9. If you get 36 on a 40-question test, what percent is this?
10. The 3200 people who vote in an election are 40% of the people registered to vote. How many are registered?

11 to 13: Your wage is increased by 20%, then the new amount is cut by 20% (of the new amount).

11. Will this result in a wage which is higher than, lower than, or the same as the original wage?
12. What percent of the original wage is this final wage?
13. If the above steps were reversed (20% cut followed by 20% increase), the final wage would be what percent of the original wage?

14 to 16: If A is increased by 25%, it equals B.
14. Which is larger, B or the original A?
15. B is what percent of A?
16. A is what percent of B?

17. What is the average of 87, 36, 48, 59, and 95?
18. If two test scores are 85 and 60, what minimum score on the next test would be needed for an overall average of 80?
19. The average height of 49 people is 68 inches. What is the new average height if a 78-inch person joins the group?

20. If the set is on for six hours every day of a 30-day month, how many kwh are used for the month?
21. If the electric company charges 8¢ per kwh, what amount of the month's bill is for TV power?

25 to 33: A plane has a certain speed in still air, where it goes 1350 miles in three hours.
25. What is its (still air) speed?
26. How far does the plane go in 5 hours?
27. How far does it go in x hours?
28. How long does it take to fly 2000 miles?
29. How long does it take to fly y miles?
30. If the plane flies against a 50 mph headwind, what is its ground speed?
31. If the plane flies against a headwind of z mph, what is its ground speed?
32. If it has fuel for 7.5 hours of flying time, how far can it go against the headwind of 50 mph?
33. If the plane has fuel for t hours of flying time, how far can it go against the headwind of z mph?
B. Algebraic Substitution and Evaluation

20 to 24: A certain TV uses 75 watts of power, and operates on 120 volts.

20. Find how many amps of current it uses, from the relationship: volts times amps equals watts.

21. 1000 watts = 1 kilowatt (kw). How many kilowatts does the TV use?

22. Kw times hours = kilowatts-hours(kwh). If the TV is on for six hours a day, how many kwh of electricity are used?

C. Ratio and Proportion

34 to 35: x is to y as 3 is to 5.

34. Find y when x is 7.

35. Find x when y is 7.

36 to 37: s is proportional to P, and P = 56 when s = 14.

36. Find s when P = 144.

37. Find P when s = 144.

38 to 39: Given 3x = 4y

38. Write the ratio x:y as the ratio of two integers.

39. If x = 3, find y.

40 to 41: x and y are numbers, and two x's equal three y's.

40. Which of x or y must be larger?

41. What is the ratio of x to y?

42 to 44: Half of x is the same as one-third of y.

42. Which of x and y is the larger?

43. Write the ratio x:y as the ratio of two integers.

44. How many x's equal 30 y's?

D. Problems leading to one linear equation

45. 36 is three-fourths of what number?

46. What number is 3/4 of 36?

47. What fraction of 36 is 15?
48. $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{3}{4}$ of a number is 12. What is the number?

49. Half the square of a number is 18. What is the number?

50. 81 is the square twice what number?

51. Given a positive number x. Two times a positive number y is at least four times x. How small can y be?

52. Twice the square root of half of a number is 2x. What is the number?

53 to 55: A gathering has twice as many women as men. W is the number of women and M is the number of men.

53. Which is correct: $2M = W$ or $M = 2W$?

54. If there are 12 women, how many men are there?

55. If the total number of men and women present is 54, how many of each are there?

56. $12,000$ is divided into equal shares. Babs gets four shares, Dill gets three shares, and Ben gets the one remaining share. What is the value of one share?

65. In order to construct a square with an area which is 100 times the area of a given square, how long a side should be used?

66 to 71: The length of a rectangle is increased by 25% and its width is decreased by 25% and its width is decreased by 40%.

66. Its new area is what percent of its old?

67. By what percent has the old area increased or decreased?

68. The length of a rectangle is twice the width. If both dimensions are increased by 2 cm, the resulting rectangle has 84 cm$^2$ more area. What was the original width?

69. After a rectangular piece of knitted fabric shrinks in length one cm and stretches in width 2 cm, it is a square. If the original area was 40 cm$^2$, what is the square area?

70. This square is cut into two smaller squares and two non-square rectangles as shown. Before being cut, the large square had area $(a + b)^2$. The two smaller squares have areas $a^2$ and $b^2$. Find the total area of the two non-square rectangles. Show that the areas of the 4 parts add up to the area of the original square.
E. Problems leading to two linear equations

57. Two science fiction coins have values x and y. Three x’s and five y’s have a value of 75¢, and one x and two y’s have a value of 27¢. What is the value of each?

58. In mixing x gm of 3% and y gm of 8% solutions to get 10 gm of 5% solution, these equations are used:

\[0.03x + 0.08y = 0.05(10),\] and
\[x + y = 10\]

How many gm of 3% solution are needed?

F. Geometry

59. Point X is on each of two given intersecting lines. How many such points X are there?

60. On the number line, points P and Q are two units apart. Q has coordinate x. What are the possible coordinates of P?

61 to 62:

61. If the length of chord AB is x and the length of CB is 16, what is AC?

62. If AC = y and CB = z, how long is AB (in terms of y and z)?

63 to 64: the base of a rectangle is three times the height.

63. Find the height if the base is 20.

64. Find the perimeter and the area.

Answers

1. 46
2. 9
3. 2
4. 1.08
5. 62.5%
6. 160%
7. 56.25
8. 5581
9. 90%
10. 8000
11. lower
12. 96%
13. same (96%)
14. B
15. 125%
16. 80%
17. 65
18. 95
19. 68.2
20. .625 amps
21. .075 kw
22. .45 kwh
23. 13.5 kwh
24. $1.08
25. 450 mph
26. 2250 mi.
27. 450 mph
28. 40/9 hr.
29. 450 hr.
30. 400 mph
31. 450-z mph
32. 3000 mi.
33. (450 - z) mi.
34. 35/3
35. 21/5
36. 36
37. 576
38. 4:3

39. 9/4
40. x
41. 3:2
42. y
43. 2:3
44. 15
45. 48
46. 27
47. 5/12
48. 144
49. 6
50. 9/2
51. 2x
52. 2x^2
53. 2M = W
54. 6
55. 18 men
56. 36 women
57. x:15¢
y: 5¢
58. 6 gm
59. 1
60. x - 2,
x + 2
61. x - 16
62. y + z
63. 20/3
64. P = 160/3
A = 400/3
65. 10 times the original size
66. 75%
67. 25% decrease
68. 40/3
69. 49
70. 2ab
\[a^2 + 2ab + b^2 = (a + b)^2\]